Wide-angle effects in the large-scale power spectrum multipoles

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Motivation

- Primordial non-Gaussianity is one of the most powerful tests of inflation we have, and LSS is going to dominate such constraints in only a few years from now:

Planck	$\lvert f_{\rm NL}\rvert < 5$
BOSS	< 23
BOSS+eBOSS	< 11
DESI	< 3.8
BOSS+Euclid	< 6.7
SPHEREx	< 1

- Galaxy surveys constrain $f_{\rm NL}$ through the large scale power spectrum which has a k^{-2} dependence.
- To allow such constraints we need to understand our measurements of the density field on the largest scales.

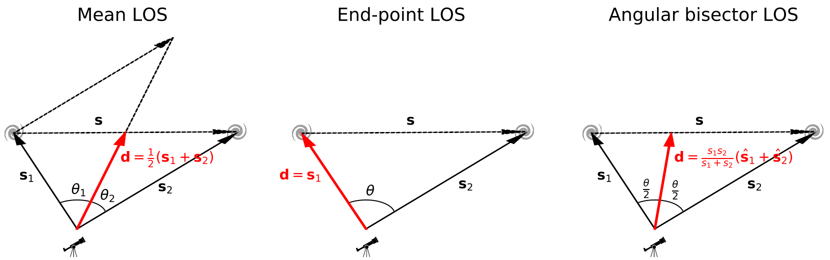
$\rightarrow$ We need to understand window function effects, wide-angle effects and the integral constraint

Window function treatment

1. Measure the window function multipoles $Q_\ell(s)$
2. Get a model for the power spectrum multipoles $P_\ell(k)$
3. Fourier transform the power spectrum model $\xi_\ell(s)$
4. Multiply the window function and correlation function multipoles
5. Fourier transform back into Fourier-space $\tilde{P}_\ell(k)$

$\rightarrow$ We can extend this formalism to include wide-angle effects

General 2-pt statistics

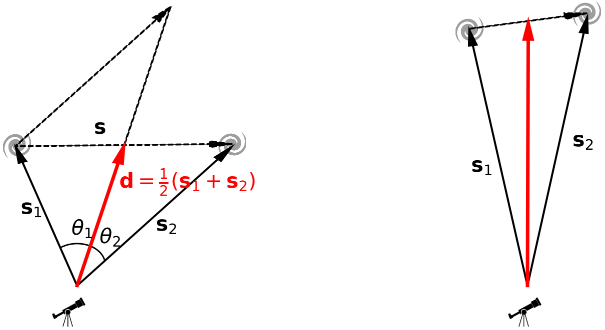


The correlation function is defined via the excess probability of finding a galaxy pair at separation s:

$$dP = \overline{n}^2 \left[1 + \xi(s)\right] dV_1 dV_2$$

RSD break isotropy and homogeneity $\rightarrow \xi(\mathbf{s}, \mathbf{d})$

Plane-parallel approximation



In the limit of large distances (small opening angles), the position vectors can be approximated as parallel to each other and he LOS choice becomes irrelevant (Kaiser 1987).

The power spectrum is now given by

$$P(\mathbf{k}, \mathbf{d}) \equiv \int \mathrm{d}^3 s \, \xi(\mathbf{s}, \mathbf{d}) e^{-i\mathbf{k}\cdot\mathbf{s}}$$

which we can expand in multipoles,

$$P(\mathbf{k}, \mathbf{d}) = \sum_{\ell} P_{\ell}(k, d) \mathcal{L}_{\ell}(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}})$$

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which we can expand in multipoles,

$$\begin{aligned} P(\mathbf{k},\mathbf{d}) &= \sum_\ell P_\ell(k,d)\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}}) \\ &\approx \sum_\ell P_\ell(k)\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}}) \end{aligned}$$

Plane-parallel approximation

The power spectrum + wide-angles

The power spectrum is now given by

$$P(\mathbf{k}, \mathbf{d}) \equiv \int \mathrm{d}^3 s \, \xi(\mathbf{s}, \mathbf{d}) e^{-i\mathbf{k}\cdot\mathbf{s}}$$

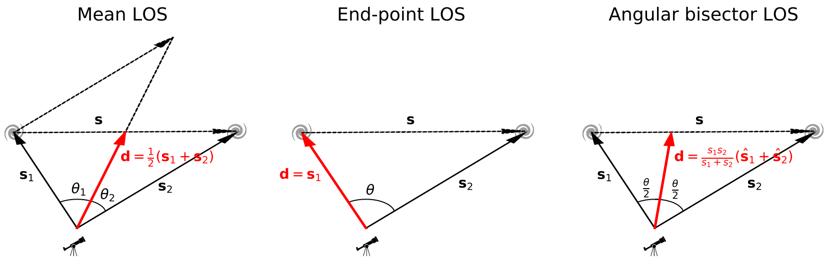
which we can expand in multipoles and wide-angle contributions,

$$\begin{aligned} P(\mathbf{k}, \mathbf{d}) &= \sum_{\ell} P_\ell(k, d) \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}}) \\ &\equiv \sum_{\ell, n} x_k^n P_\ell^{(n)}(k) \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}}), \end{aligned}$$

with $x_k = (kd)^{-1}$. This follows the Hankel transform

$$P_\ell^{(n)}(k) = 4\pi(-i)^\ell \int s^2 \, \mathrm{d}s \, (ks)^n \, \xi_\ell^{(n)}(s) \, j_\ell(ks)$$

Odd multipoles



- All FFT based estimators require the end-point LOS, which breaks the symmetry between the galaxy pair.

$$\xi_1^{\mathrm{ep},(n=1)}(s) = -\frac{3}{5}\xi_2^{(n=0)}(s),$$

$$\xi_3^{\mathrm{ep},(n=1)}(s) = \frac{3}{5}\xi_2^{(n=0)}(s) - \frac{10}{9}\xi_4^{(n=0)}(s)$$

- In Fourier-space the odd multipoles couple to the even multipoles through the survey window.

Power spectrum estimate and modelling

The most popular estimator is (Yamamoto et al. 2005)

$$P_\ell^{ep}(k) = \left\langle \frac{(2\ell+1)}{2A} \int d\mathbf{s}_1 \int d\mathbf{s}_2\, \delta_g(\mathbf{s}_1)\delta_g(\mathbf{s}_2) e^{i\mathbf{k}\cdot(\mathbf{s}_1-\mathbf{s}_2)} \mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{s}}_1) - S_\ell \right\rangle$$

To model this power spectrum we need to include the survey geometry

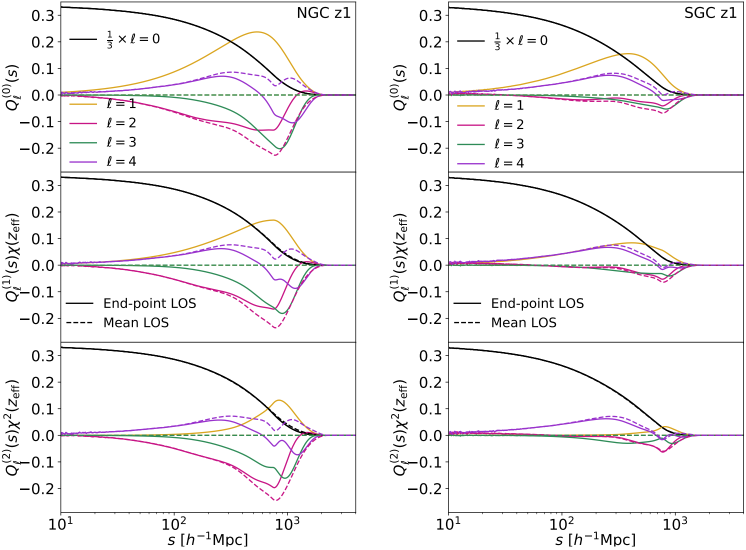
$$Q_L^{ep,(n)}(s) \equiv (2L+1) \int \mathrm{d}\Omega_s \int \mathrm{d}^3 s_1 (s_1)^{-n} W(\mathbf{s}_1) W(\mathbf{s}+\mathbf{s}_1) \mathcal{L}_L(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}_1)$$

which couples to the multipoles as

$$\left\langle \tilde{P}_A(\mathbf{k}) \right\rangle = (-i)^A (2A+1) \sum_{\ell,\,L} \begin{pmatrix} \ell & L & A \\ 0 & 0 & 0 \end{pmatrix}^2 \int \mathrm{d}s\, j_A(ks)$$

$$\sum_n s^{n+2}\, \xi_\ell^{\mathrm{ep},(n)}(s) Q_L^{\mathrm{ep},(n)}(s)$$

Survey window function



Convolved model

Monopole:

$$\tilde{\xi}_0^{(n)}(s) = \xi_0^{(n)}(s)Q_0^{(n)}(s) + \frac{1}{5}\xi_2^{(n)}(s)Q_2^{(n)}(s) + \frac{1}{9}\xi_4^{(n)}(s)Q_4^{(n)}(s)$$

$$\tilde{\xi}_0^{(1)}(s) = \frac{1}{3}\xi_1^{(1)}(s)Q_1^{(1)}(s) + \frac{1}{7}\xi_3^{(1)}(s)Q_3^{(1)}(s) + \cdots.$$

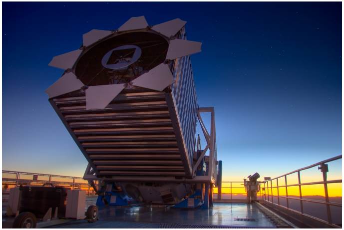
Dipole:

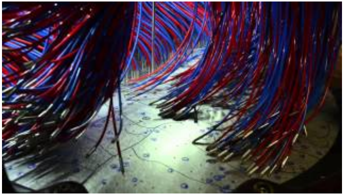
$$\tilde{\xi}_1^{(0)}(s) = \xi_0^{(0)}(s)Q_1^{(0)}(s) + \xi_2^{(0)}(s)\left[\frac{2}{5}Q_1^{(0)}(s) + \frac{9}{35}Q_3^{(0)}(s)\right] + \xi_4^{(0)}(s)\left[\frac{4}{21}Q_3^{(0)}(s) + \frac{5}{33}Q_5^{(0)}(s)\right]$$

$$\tilde{\xi}_1^{(1)}(s) = \xi_1^{(1)}(s)\left[Q_0^{(1)}(s) + \frac{2}{5}Q_2^{(1)}(s)\right] + \xi_3^{(1)}(s)\left[\frac{9}{35}Q_2^{(1)}(s) + \frac{4}{21}Q_4^{(1)}(s)\right] + \cdots$$

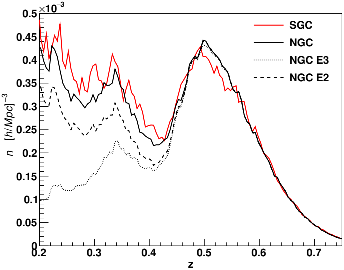
The BOSS galaxy survey

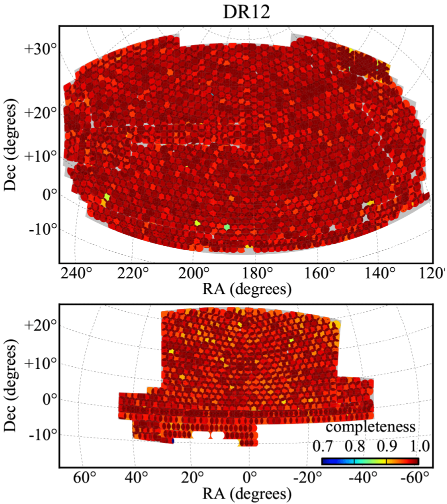
- Third version of the Sloan Digital Sky Survey (SDSS-III).
- Spectroscopic survey optimized for the measurement of Baryon Acoustic Oscillations (BAO).
- The galaxy sample includes 1 100 000 galaxy redshifts in the range $0.2 < z < 0.75$.
- The effective volume is $\sim 6\,\text{Gpc}^3$.
- 1000 fibres/pointing



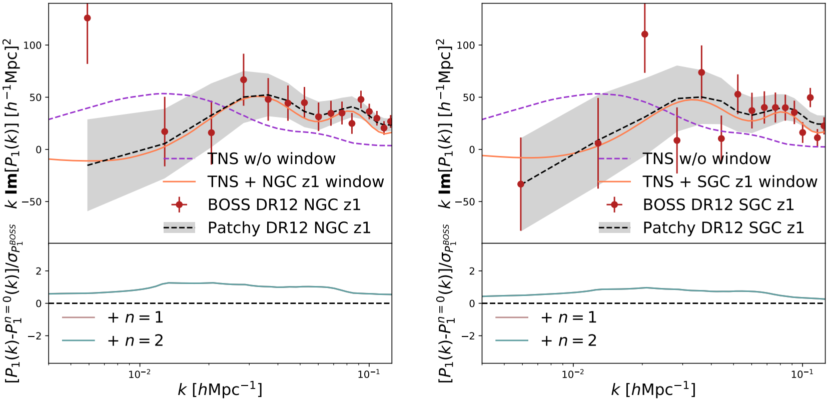


- The final data release (DR12) covers about 10 000 deg^2.
- The survey is divided in a north galactic patch (NGC) and a south galactic patch (SGC).





Power spectrum dipole in BOSS DR12



- The dipole can be detected with $> 10\sigma$ in each redshift bin!
- A re-analysis of BOSS RSD and BAO did not show a significant impact
- Understanding this dipole is essential to be able to measure the relativistic dipole

Integral constraint

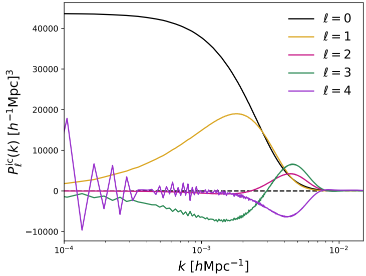
The mean density of the universe is assumed to be equal to the mean density in the survey

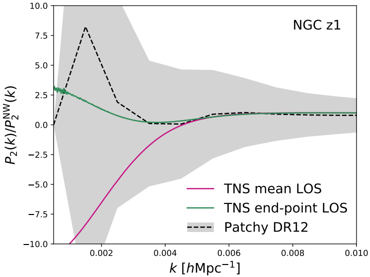
$$\delta(\mathbf{x}) = \frac{n(\mathbf{x})}{\overline{n}} - 1$$

where $\overline{n}$ is the mean density in the survey. This forces the power spectrum to zero on the largest scales. We have to include this in the model

$$P^{\rm ic}_\ell(k) = \tilde{P}_0(0)Q^{(0)}_\ell(k)$$

and $\tilde{P}_\ell(k) = \tilde{P}_\ell(k) - P^{\rm ic}_\ell(k)$

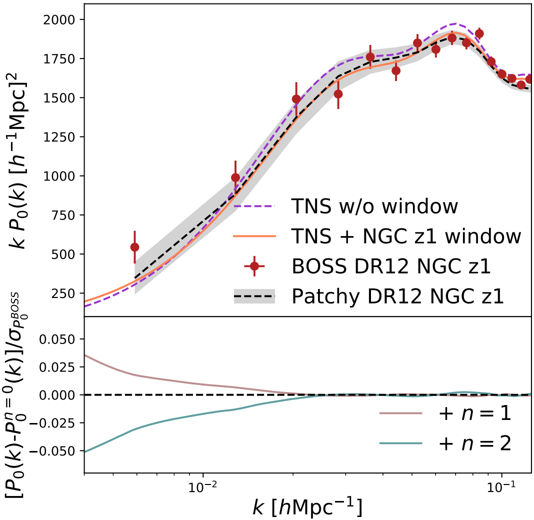




Conclusion

1. Wide-angle effects depend on the choice of the LOS. FFT-based estimators use the end-point LOS, which introduces odd multipoles.
2. The odd multipoles couple to the even multipoles through the survey window.
3. The dipole can be detected with $> 10\sigma$ in both redshift bins of BOSS.
4. Since it impacts large scales, it mainly matters for $f_{\rm NL}$ constraints.
5. It also matters for observables which show up in the odd multipoles, like e.g. the relativistic dipole.
6. Our formalism can account for all these effects in the power spectrum model without any additional free parameters.

Monopole



2000
1750
1500
1250
1000
750
500
250
$k\, P_0(k)\ [h^{-1}\mathrm{Mpc}]^2$
TNS w/o window
TNS + NGC z1 window
BOSS DR12 NGC z1
Patchy DR12 NGC z1
0.050
0.025
0.000
−0.025
−0.050
$[P_0(k)\text{-}P_0^{n=0}(k)]/\sigma_{P_0^{BOSS}}$
$+\ n = 1$
$+\ n = 2$
10^{-2}
10^{-1}
$k\ [h\mathrm{Mpc}^{-1}]$