Unified galaxy power spectrum measurements from 6dFGS, BOSS, and eBOSS – arXiv:2106.06324

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- In Fourier-space we have to deal with two complications when performing a data analysis:
  1. Window function
  2. Wide-angle effects
- We can deal with these complications by:
  1. convolving the model or
  2. deconvolving the data

Window function

- To get to the power spectrum ($P(k) = \langle\delta^2\rangle$) we need to measure the overdensity field
- With galaxy surveys we cannot measure the overdensity field directly but rather $W(x)\delta(x)$
- In Fourier-space this multiplication becomes a convolution, so that the power spectrum measured with a galaxy survey is

$$P^{\mathrm{conv}}(k) = \int d^3k' P^{\mathrm{true}}(k')|W(k-k')|^2$$

- We now have two options:
  1. Convolve the model power spectrum before comparing to the data or
  2. Deconvolve the measured power spectrum

Window function – convolving the model

The current standard approach is (Wilson et al. 2015):

1. Measure the window function $W(x)$. The window function is the two-point statistic of the random catalog
2. Fourier transform the power spectrum model into configuration space

$$\xi_\ell(x) = i^\ell \int \frac{k^2 dk}{2\pi^2} P_\ell(k) j_\ell(ks)$$

3. Multiply the configuration-space model with $Q_\ell(x) = W_\ell^2(x)$

$$\xi_0^{\rm conv} = \xi_0 Q_0 + \frac{1}{5}\xi_2 Q_2 + \frac{1}{9}\xi_4 Q_4 + \dots$$

4. Fourier-transform back into Fourier-space
5. Compare to the measured power spectrum and calculate χ^2...

Window function – alternative approach

Alternatively, one can perform the convolution directly in Fourier-space (Beutler et al. 2014):

$$P_\ell^{\rm conv}(k) = \int dk' k'^2 \sum_{\ell'} W_{\ell\ell'}(k,k') P_{\ell'}^{\rm true}(k')$$

which requires to measure (D'Amico et al. 2020)

$$W_{\ell\ell'}(k,k') = \frac{2}{\pi}(-i)^\ell i^{\ell'} \int ds\, s^2 j_\ell(ks) j_{\ell'}(k's) \sum_L C_{\ell\ell' L} W_L(s)$$

With this we can define a matrix based window function approach

Window function – matrix based approach

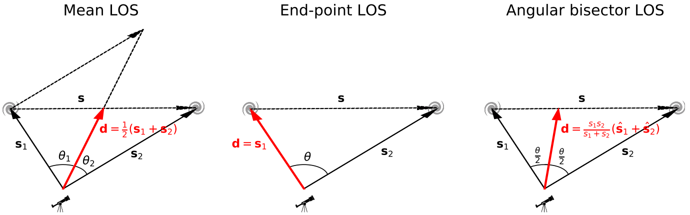
$$\mathbf{P}^{\rm true,flat-sky} = \begin{pmatrix} P_0^{\rm true}(k) \\ P_2^{\rm true}(k) \\ P_4^{\rm true}(k) \end{pmatrix}, \quad \mathbf{P}^{\rm conv,flat-sky} = \begin{pmatrix} P_0^{\rm conv}(k) \\ P_2^{\rm conv}(k) \\ P_4^{\rm conv}(k) \end{pmatrix}$$

$$\mathbf{W} = \begin{pmatrix} W_{00} & W_{02} & W_{04} \\ W_{20} & W_{22} & W_{24} \\ W_{40} & W_{42} & W_{44} \end{pmatrix}$$

With this we can calculate the convolved power spectrum as

$$\mathbf{P}^{\rm conv,flat-sky} = \mathbf{W}\mathbf{P}^{\rm true,flat-sky}$$

Wide-angle effects



The choice of a line-of-sight (LOS) in the estimator will introduce wide-angle effects (Yamamoto et al. 2006)

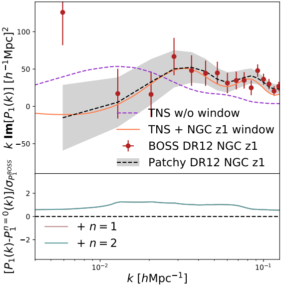
$$P_\ell(k) = \frac{2\ell + 1}{2A} \int d\vec{x}_1 \int d\vec{x}_2 F(\vec{x}_1) F(\vec{x}_2) e^{i\mathbf{k}\cdot(\mathbf{x}_1 - \mathbf{x}_2)} \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}}) - S$$

We can include wide-angle effects in our power spectrum model (Reimberg et al. 2016)

$$P(\mathbf{k}, \mathbf{d}) = \sum_\ell P_\ell(k, d) \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}}) = \sum_{\ell, n} \left(\frac{1}{kd}\right)^n P_\ell^{(n)}(k) \mathcal{L}_\ell(\hat{\mathbf{k}} \cdot \hat{\mathbf{d}})$$

Wide-angle effects

The end-point LOS can introduce odd multipoles (Beutler et al. 2019)



And they can be calculated directly from the even multipoles:

$$P_1^{(1)}(k) = -i\frac{3}{5}\left[\frac{3}{k}P_2^{(0)}(k) + \partial_k P_2^{(0)}(k)\right]$$

$$P_3^{(1)}(k) = -i\left[\frac{3}{5}\left(\frac{2}{k}P_2^{(0)}(k) - \partial_k P_2^{(0)}(k)\right) + \frac{10}{9}\left(\frac{5}{k}P_4^{(0)}(k) + \partial_k P_4^{(0)}(k)\right)\right]$$

Wide-angle effects in matrix form

$$\mathbf{M} = \begin{pmatrix} \mathcal{I} & 0 & 0 \\ 0 & K^{2\to1} & 0 \\ 0 & \mathcal{I} & 0 \\ 0 & K^{2\to3} & K^{4\to3} \\ 0 & 0 & \mathcal{I} \end{pmatrix} \qquad \mathbf{P}^{\rm true} = \mathbf{M}\mathbf{P}^{\rm true,flat-sky}$$

where $\mathcal{I}$ is the identity matrix and

$$K^{2\to1}_{lm} = -i\frac{3}{5d}\left[\frac{3}{k_l}\Theta(k_l,k_m) + \partial_{k_m}\Theta(k_l,k_m)\right]$$

$$= -i\frac{3}{5d}\left[\frac{3}{k_l}\Theta(k_l,k_m) + \frac{\Theta(k_l,k_m-\Delta k)-\Theta(k_l,k_m+\Delta k)}{2\Delta k}\right]$$

$$K^{2\to3}_{lm} = -i\frac{3}{5d}\left[\frac{2}{k_l}\Theta(k_l,k_m) - \frac{\Theta(k_l,k_m-\Delta k)-\Theta(k_l,k_m+\Delta k)}{2\Delta k}\right]$$

$$K^{4\to3}_{lm} = -i\frac{10}{9d}\left[\frac{5}{k_l}\Theta(k_l,k_m) + \frac{\Theta(k_l,k_m-\Delta k)-\Theta(k_l,k_m+\Delta k)}{2\Delta k}\right]$$

The code to calculate this matrix is on github

https://github.com/fbeutler/pk_tools/blob/master/wide_angle_tools.py

This introduces two problems:

1. All multipoles are connected through the window function... so the dipole can impact the monopole and quadrupole and ignoring it could bias our RSD and BAO measurements
2. The dipole and octopole carry information on the anisotropy which would need to be included for any **optimal** analysis

We can include the wide-angle effects in the window function

$$W^{(n)}_{\ell\ell'}(k,k') = \frac{2}{\pi}(-i)^{\ell} i^{\ell'} \int ds\, s^2 j_{\ell}(ks) j_{\ell'}(k's) \sum_L C^{(n)}_{\ell\ell' L} Q^{(n)}_L(s)$$

and power spectrum convolution

$$P^{\rm conv}_{\ell}(k) = \int dk' \sum_{\ell',n} k'^{2-n} W^{(n)}_{\ell\ell'}(k,k') P^{(n),\rm true}_{\ell'}(k')$$

Window function – matrix based approach

$$\mathbf{P}^{\rm true,flat-sky} = \begin{pmatrix} P_0^{(0),\rm true}(k) \\ P_2^{(0),\rm true}(k) \\ P_4^{(0),\rm true}(k) \end{pmatrix}, \quad \mathbf{P}^{\rm true} = \begin{pmatrix} P_0^{(0),\rm true}(k) \\ P_1^{(1),\rm true}(k) \\ P_2^{(0),\rm true}(k) \\ P_3^{(1),\rm true}(k) \\ P_4^{(0),\rm true}(k) \end{pmatrix}, \quad \mathbf{P}^{\rm conv} = \begin{pmatrix} P_0^{(0),\rm conv}(k) \\ P_1^{(1),\rm conv}(k) \\ P_2^{(0),\rm conv}(k) \\ P_3^{(1),\rm conv}(k) \\ P_4^{(0),\rm conv}(k) \end{pmatrix}$$

$$\mathbf{W} = \begin{pmatrix} W_{00}^{(0)} & W_{01}^{(1)} & W_{02}^{(0)} & W_{03}^{(1)} & W_{04}^{(0)} \\ W_{10}^{(0)} & W_{11}^{(1)} & W_{12}^{(0)} & W_{13}^{(1)} & W_{14}^{(0)} \\ W_{20}^{(0)} & W_{21}^{(1)} & W_{22}^{(0)} & W_{23}^{(1)} & W_{24}^{(0)} \\ W_{30}^{(0)} & W_{31}^{(1)} & W_{32}^{(0)} & W_{33}^{(1)} & W_{34}^{(0)} \\ W_{40}^{(0)} & W_{41}^{(1)} & W_{42}^{(0)} & W_{43}^{(1)} & W_{44}^{(0)} \end{pmatrix}$$

With this we can calculate the convolved power spectrum as

$$\mathbf{P}^{\rm conv} = \mathbf{W}\mathbf{M}\mathbf{P}^{\rm true,flat-sky}$$

Summary – matrix formalism

1. Get the standard multipoles in the flat-sky approximation $\mathbf{P}^{\text{true,flat-sky}} = (P_0, P_2, P_4)$
2. Multiply with $\mathbf{M}$ to get $\mathbf{P}^{\text{true}} = (P_0, P_1, P_2, P_3, P_4)$
3. Multiply with $\mathbf{W}$ to get $\mathbf{P}^{\text{conv}} = (P_0, P_1, P_2, P_3, P_4)$
4. compare with data

$$\mathcal{L} \sim \exp\left[-\frac{1}{2}(\mathbf{P}_{\text{o}}^{\text{conv}} - \mathbf{WMP}^{\text{true,flat-sky,T}})\mathbf{C}_{\text{conv}}^{-1}(\mathbf{P}_{\text{o}}^{\text{conv}} - \mathbf{WMP}^{\text{true,flat-sky}})\right]$$

What about deconvolution?

Starting from the likelihood

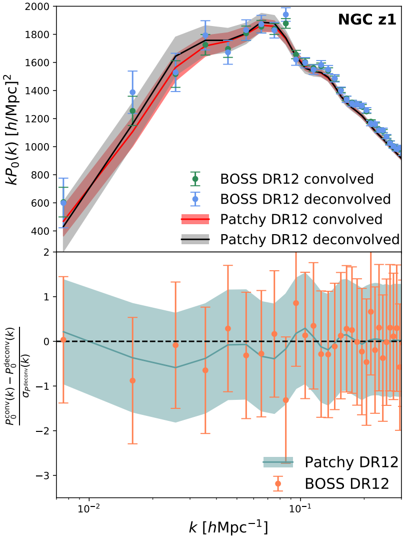
$$\mathcal{L} \sim \exp\left[-\frac{1}{2}(\mathbf{P}_{\rm o}^{\rm conv} - \mathbf{W}\mathbf{P}^{\rm true,flat\text{-}sky,T})\mathbf{C}_{\rm conv}^{-1}(\mathbf{P}_{\rm o}^{\rm conv} - \mathbf{W}\mathbf{P}^{\rm true,flat\text{-}sky})\right]$$

and the maximum likelihood condition $\partial\chi^2/\partial\mathbf{P}^{\rm true} = 0$ one can derive

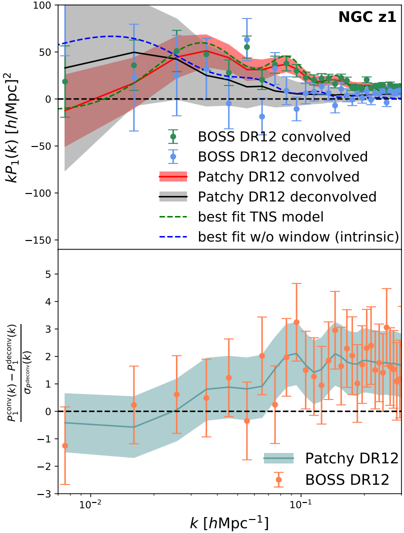
$$(\mathbf{W}^T\mathbf{C}_{\rm conv}^{-1}\mathbf{W})^{-1}\mathbf{W}^T\mathbf{C}_{\rm conv}^{-1}\mathbf{P}_{\rm o}^{\rm conv} \equiv \mathbf{P}_{\rm o}^{\rm true}$$

which allows to deconvolve the measured power spectrum

BOSS deconvolved power spectrum monopole



BOSS deconvolved power spectrum dipole



Summary

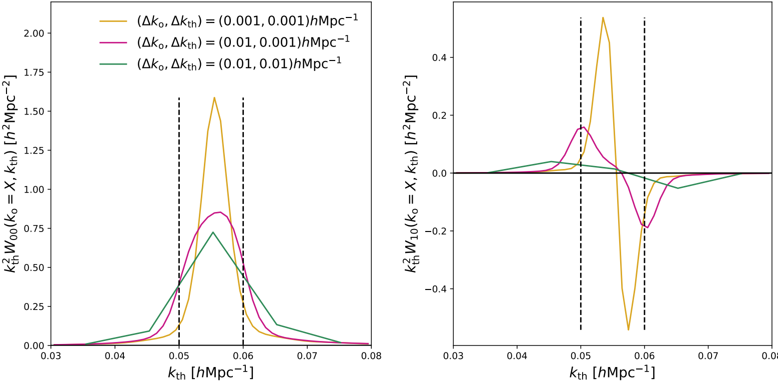
1. We should make **W** and **M** available with the DESI catalogs $\longrightarrow$ can be adopted easily (k-binning choices?)
2. Deconvolution might have some use-cases, but generally the model convolution approach seems superior
3. You can get all these products for 6dFGS, BOSS and eBOSS-QSO here – `https://fbeutler.github.io/hub/deconv_paper.html`

k-bin average

$$W^{(n)}_{\ell\ell'}(k_{\rm o}, k_{\rm th}) = \int dk\, k^2 \Theta(k_{\rm o}, k) \int dk'\, k'^{2-n} W^{(n)}_{\ell\ell'}(k, k')\Theta(k_{\rm th}, k')$$

with

$$\Theta(k_{\rm x}, k) = \begin{cases} 1 & \text{if } |k_x - k| < \frac{\Delta k_{\rm x}}{2} \\ 0 & \text{otherwise.} \end{cases}$$



1. We will end all telecons 10 mins earlier (9:50 PDT), reserving the last 10 min for an (informal) chat about opportunities to get involved or Q&A with WG chairs & KP leads.
2. We will try out slido (https://www.sli.do/) to ask (and vote for) questions (anonymously if preferred) during talks. The slido link will be sent with the telecon announcement.
3. We will use one telecon every 2 months for KP leads, MC leads (and others) to present 'open tasks' and 'ways to get started with DESI data/mocks' to increase the engagement with WG activities.