A new decomposition formalism for the bispectrum

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Towards the full analysis of the bispectrum in redshift space I: a new decomposition formalism

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ABSTRACT

We propose a new decomposition formalism for computing the anisotropic bispectrum in redshift space and for measuring it from galaxy samples. Via the decomposition into the tri-polar spherical harmonic basis with zero total angular momentum, the signal induced by redshift space distortions (RSDs) can be completely distinguished from

Decomposition formalism

We propose to decompose the Bispectrum in spherical harmonics in $\hat{k}_1$, $\hat{k}_2$ and the los $\hat{n}$:

$$B_{\ell_1\ell_2 L}(k_1, k_2) = H_{\ell_1\ell_2 L} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} B^{m_1 m_2 M}_{\ell_1\ell_2 L}(k_1, k_2).$$

with

$$H_{\ell_1\ell_2 L} = \begin{pmatrix} \ell_1 & \ell_2 & L \\ 0 & 0 & 0 \end{pmatrix}$$

- The summation over the azimuthal angles is possible because of isotropy and allowed multipoles have to follow the relation $\ell_1 + \ell_2 + L =$ even.
- These bispectrum multipoles contain all physical information under the three statistical assumptions: homogeneity, isotropy, and parity-symmetry of the Universe.

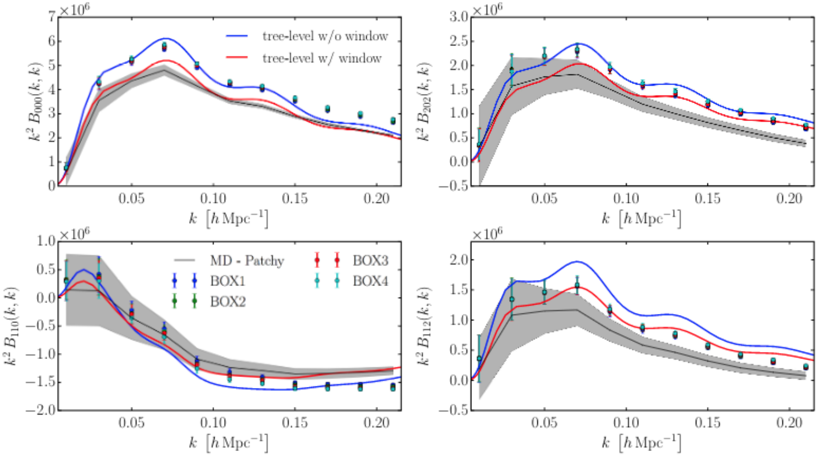
Why using this formalism (comparison to Scoccimarro 2015)

Scoccimarro (2015) decomposes in $\hat{k}_1$:

$$B_{\ell m}(k_1, k_2, k_3) = \frac{2\ell + 1}{N^T_{123}} \prod_{i=1}^{3} \int_{k_i} d^3 q_i \delta_D(q_{123}) \delta_\ell(q_1) \delta_0(q_2) \delta_0(q_3)$$

- Our decomposition allows for a self consistent inclusion of the window function.
- The decomposition in two k vectors is more practical because of the closed triangle condition. There is no need to enforce this condition after the bispectrum estimation.
- The RSD information is clearly separated into the L multipoles.
- The complexity of our estimator is $\mathcal{O}((2\ell_1 + 1) N_b^2 N \log N)$ compared to $\mathcal{O}(N_b^3 N \log N)$ in Scoccimarro 2015 (however, the closed triangle condition reduces Roman's estimator complexity effectively to $\mathcal{O}(N_b^2 N \log N)$).

Accounting for the survey window



Appendix: Accounting for the survey window

We can estimate the survey window very similar to the bispectrum estimator

$$
\begin{aligned}
Q_{\ell_1\ell_2 L}(r_1, r_2) = H_{\ell_1\ell_2 L} \sum_{m_1 m_2 M} & \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} \\
& \times \frac{N_{\ell_1\ell_2 L}}{I} \int \frac{d^2\hat{r}_1}{4\pi} y_{\ell_1}^{m_1 *}(\hat{r}_1) \int \frac{d^2\hat{r}_2}{4\pi} y_{\ell_2}^{m_2 *}(\hat{r}_2) \\
& \times \int d^3x_1 \int d^3x_2 \int d^3x_3 \\
& \times \delta_{\rm D}\left(\vec{r}_1 - \vec{x}_{13}\right) \delta_{\rm D}\left(\vec{r}_2 - \vec{x}_{23}\right) \\
& \times y_L^{M*}(\hat{x}_3)\, \bar{n}(\vec{x}_1)\, \bar{n}(\vec{x}_2)\, \bar{n}(\vec{x}_3).
\end{aligned}
$$

We can now follow the steps of Wilson et al. (2015)/Beutler et al. (2017) to include the window function in the analysis pipeline.

1. Hankel transform to the three-point function
2. Multiply with the window function
3. Hankel transform back

Step 1 & step 3: The Hankel transform for the bispectrum - three point function is given by

$$B_{\ell_1\ell_2L}(k_1,k_2) = (-i)^{\ell_1+\ell_2}(4\pi)^2\int dr_1 r_1^2 \int dr_2 r_2^2 \times \; j_{\ell_1}(k_1 r_1) j_{\ell_2}(k_2 r_2)\zeta_{\ell_1\ell_2L}(r_1,r_2)$$

$$\zeta_{\ell_1\ell_2L}(r_1,r_2) = i^{\ell_1+\ell_2}\int \frac{dk_1 k_1^2}{2\pi^2}\int \frac{dk_2 k_2^2}{2\pi^2} \times \; j_{\ell_1}(r_1 k_1) j_{\ell_2}(r_2 k_2) B_{\ell_1\ell_2L}(k_1,k_2),$$

Appendix: Accounting for the survey window

Step 2: Multiply the three-point function with the survey window

$$\left\langle \widehat{\zeta}_{\ell_1\ell_2 L}(r_1, r_2) \right\rangle$$
$$= N_{\ell_1\ell_2 L} \sum_{\ell_1'+\ell_2'+L'=\mathrm{even}} \sum_{\ell_1''+\ell_2''+L''=\mathrm{even}}$$
$$\times \begin{Bmatrix} \ell_1'' & \ell_2'' & L'' \\ \ell_1' & \ell_2' & L' \\ \ell_1 & \ell_2 & L \end{Bmatrix} \left[\frac{H_{\ell_1\ell_2 L} H_{\ell_1\ell_1'\ell_1''} H_{\ell_2\ell_2'\ell_2''} H_{LL'L''}}{H_{\ell_1'\ell_2'L'} H_{\ell_1''\ell_2''L''}} \right]$$
$$\times\, Q_{\ell_1''\ell_2''L''}(r_1, r_2)\, \zeta_{\ell_1'\ell_2'L'}(r_1, r_2)$$
$$-\quad Q_{\ell_1\ell_2 L}(r_1, r_2)\, \bar{\zeta},$$

Appendix: The estimator in detail

The estimator is based on the spherical harmonics expansion proposed in Sugiyama et al. (2017), Hand et al. (2017)

$$\widehat{B}_{\ell_1\ell_2L}(k_1,k_2) = H_{\ell_1\ell_2L}\sum_{m_1m_2M}\begin{pmatrix}\ell_1 & \ell_2 & L\\ m_1 & m_2 & M\end{pmatrix}$$
$$\times\frac{N_{\ell_1\ell_2L}}{I}\int\frac{d^2\hat{k}_1}{4\pi}y_{\ell_1}^{m_1*}(\hat{k}_1)\int\frac{d^2\hat{k}_2}{4\pi}y_{\ell_2}^{m_2*}(\hat{k}_2)$$
$$\times\int\frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)$$
$$\times\,\delta n(\vec{k}_1)\,\delta n(\vec{k}_2)\,\delta n_L^M(\vec{k}_3)$$

were y_L^{M*}-weighted density fluctuation

$$\delta n_L^M(\vec{x}) \equiv y_L^{M*}(\hat{x})\,\delta n(\vec{x})$$
$$\delta n_L^M(\vec{k}) = \int d^3x\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\delta n_L^M(\vec{x})$$

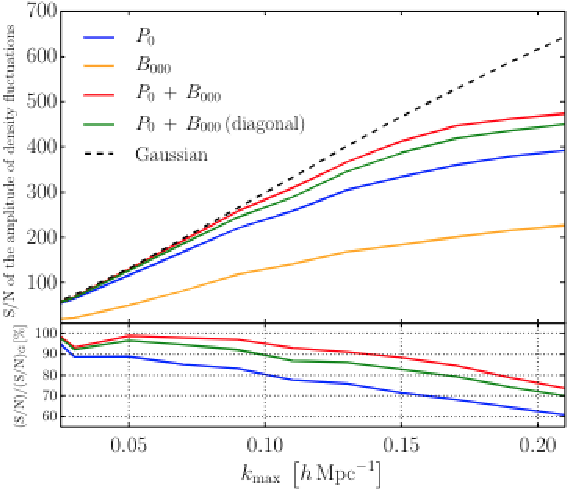
and $y_\ell^m = \sqrt{4\pi/(2\ell+1)}\,Y_\ell^m$.

Appendix: It also works for the three-point function

We can apply the same formalism to the three-point function

$$\zeta_{\ell_1\ell_2L}(r_1, r_2) = H_{\ell_1\ell_2L} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} \zeta^{m_1 m_2 M}_{\ell_1\ell_2L}(r_1, r_2).$$

Appendix: Signal to Noise, Recovering the Gaussian information level



Appendix: Relation to other decompositions

Transformation between Scoccimarro (2015) and our decomposition

$$B_{\ell_1\ell_2L}(k_1,k_2) = \frac{N_{\ell_1\ell_2L}H_{\ell_1\ell_2L}}{\sqrt{(4\pi)(2L+1)}}\int\frac{d\cos\theta_{12}}{2}$$
$$\times\left[\sum_M \begin{pmatrix}\ell_1 & \ell_2 & L\\ 0 & -M & M\end{pmatrix} y_{\ell_2}^{-M*}(\cos\theta_{12},\pi/2)\right]\times B_{LM}(k_1,k_2,\theta_{12})$$

Transformation between Slepian & Eisenstein (2017) and our decomposition:

$$B_{\ell_1\ell_2L}(k_1,k_2) = N_{\ell_1\ell_2L}H_{\ell_1\ell_2L}\sum_m(-1)^m \begin{pmatrix}\ell_1 & \ell_2 & L\\ m & -m & 0\end{pmatrix}$$
$$\times\sqrt{\frac{(\ell_1-|m|)!}{(\ell_1+|m|)!}}\sqrt{\frac{(\ell_2-|m|)!}{(\ell_2+|m|)!}}$$
$$\times\int\frac{d\cos\theta_1 d\varphi_{12}}{4\pi}\int\frac{d\cos\theta_2}{2}$$
$$\times\cos(m\varphi_{12})\mathcal{L}_{\ell_1}^{|m|}(\cos\theta_1)\mathcal{L}_{\ell_2}^{|m|}(\cos\theta_2)\times B(k_1,k_2,\theta_1,\theta_2,\varphi_{12})$$