A new decomposition formalism for the bispectrum

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Towards the full analysis of the bispectrum in redshift space I: a new decomposition formalism

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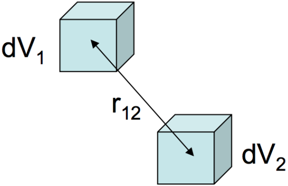
ABSTRACT

We propose a new decomposition formalism for computing the anisotropic bispectrum in redshift space and for measuring it from galaxy samples. Via the decomposition into the tri-polar spherical harmonic basis with zero total angular momentum, the signal induced by redshift space distortions (RSDs) can be completely distinguished from

From a point distribution to a power spectrum



- Overdensity-field:

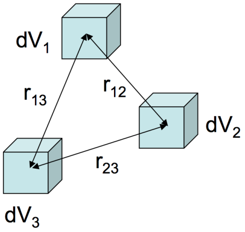
$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \overline{\rho}}{\overline{\rho}}$$

- Two-point function:

$$\xi(\mathbf{r}) \overset{\text{homogeneity}}{=} \langle\delta(\mathbf{x}+\mathbf{r})\delta(\mathbf{x})\rangle \begin{cases} \overset{\text{isotropy}}{=} & \xi(r) \\ \overset{\text{anisotropy}}{=} & \xi_\ell(r) = \int_{-1}^{1} d\mu\, \xi(r,\mu)\mathcal{L}_\ell(\mu) \end{cases}$$

- ...and in Fourier-space:

$$P_\ell(k) = 4\pi(-i)^\ell \int r^2 dr \xi_\ell(r) j_\ell(kr)$$



- Overdensity-field:

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \overline{\rho}}{\overline{\rho}}$$

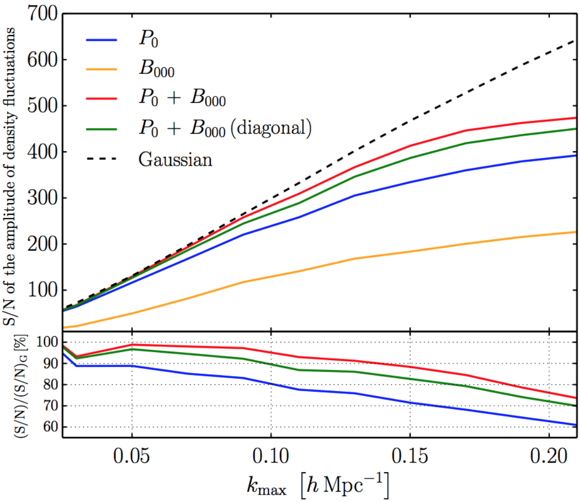
- Three-point function:

$$\xi(\mathbf{r}_1, \mathbf{r}_2) \overset{\text{homogeneity}}{=} \langle \delta(\mathbf{x}+\mathbf{r}_1)\delta(\mathbf{x}+\mathbf{r}_2)\delta(\mathbf{x})\rangle \begin{cases} \overset{\text{isotropy}}{=} & \xi_L(r_1, r_2) \\ \overset{\text{anisotropy}}{\rightarrow} & \xi_{\ell_1\ell_2L}(r_1, r_2) \end{cases}$$

- ...and in Fourier-space:

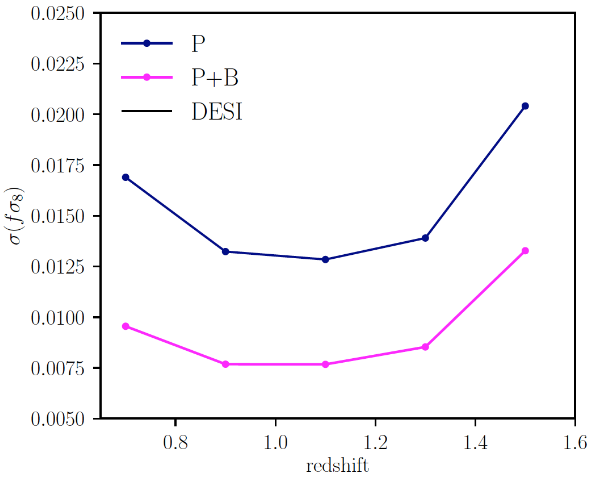
$$B_{\ell_1\ell_2L}(k_1, k_2) = (4\pi)^2(-i)^{\ell_1+\ell_2}\int r_1^2 dr_1 \int r_2^2 dr_2 \xi_{\ell_1\ell_2L}(r_1, r_2) j_{\ell_1}(k_1 r_1) j_{\ell_2}(k_2 r_2)$$

Why the bispectrum?



Sugiyama et al. (2018)

Why the bispectrum? (preliminary)



Sugiyama et al. (in prep.)

We propose to decompose the Bispectrum in spherical harmonics in $\hat{k}_1$, $\hat{k}_2$ and the los $\hat{n}$:

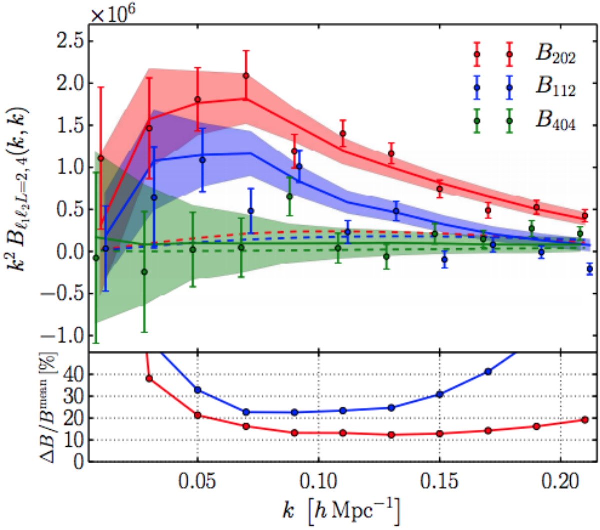
$$B_{\ell_1\ell_2L}(k_1,k_2) = H_{\ell_1\ell_2L}\sum_{m_1m_2M}\begin{pmatrix}\ell_1 & \ell_2 & L\\ m_1 & m_2 & M\end{pmatrix} B^{m_1m_2M}_{\ell_1\ell_2L}(k_1,k_2).$$

with

$$H_{\ell_1\ell_2L} = \begin{pmatrix}\ell_1 & \ell_2 & L\\ 0 & 0 & 0\end{pmatrix}$$

- The summation over the azimuthal angles is possible because of isotropy and any non-zero multipole has to follow the relation $\ell_1+\ell_2+L=$ even.
- These bispectrum multipoles contain all physical information under the three statistical assumptions: homogeneity, isotropy, and parity-symmetry of the Universe.

First measurement of the anisotropic bispectrum



Sugiyama et al. (2018)

Why using this formalism (comparison to Scoccimarro 2015)

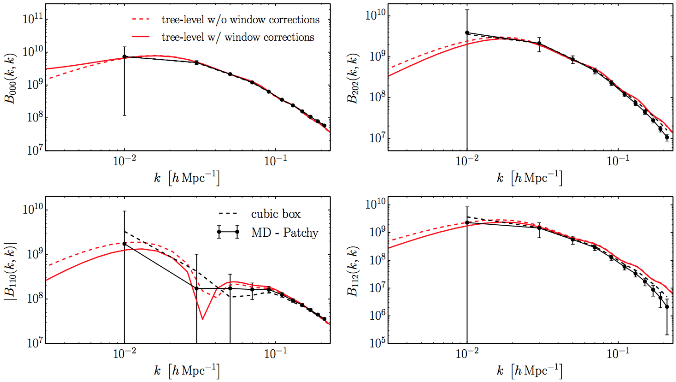
Scoccimarro (2015) decomposes in $\hat{k}_1$:

$$B_{\ell m}(k_1, k_2, k_3) = \frac{2\ell + 1}{N^T_{123}} \prod_{i=1}^{3} \int_{k_i} d^3 q_i \delta_D(q_{123}) \delta_\ell(q_1) \delta_0(q_2) \delta_0(q_3)$$

- Our decomposition allows for a self consistent inclusion of the window function.
- The decomposition in two k vectors is more practical because of the closed triangle condition. There is no need to enforce this condition after the bispectrum estimation.
- The RSD information is clearly separated into the L multipoles.
- The complexity of our estimator is $\mathcal{O}((2\ell_1 + 1)N_b^2 N \log N)$ compared to $\mathcal{O}(N_b^3 N \log N)$ in Scoccimarro 2015 (however, the closed triangle condition reduces Roman's estimator complexity effectively to $\mathcal{O}(N_b^2 N \log N)$).

Accounting for the survey window

1. Henkel transform the bispectrum (into three-point function)
2. multiply with the window function
3. Henkel transform back into FT space



Sugiyama et al. (2018)

Step 1 & step 3: The Hankel transform for the bispectrum - three point function is given by

$$B_{\ell_1\ell_2L}(k_1,k_2) = (-i)^{\ell_1+\ell_2}(4\pi)^2\int dr_1 r_1^2\int dr_2 r_2^2$$
$$\times\quad j_{\ell_1}(k_1r_1)j_{\ell_2}(k_2r_2)\zeta_{\ell_1\ell_2L}(r_1,r_2)$$
$$\zeta_{\ell_1\ell_2L}(r_1,r_2) = i^{\ell_1+\ell_2}\int\frac{dk_1k_1^2}{2\pi^2}\int\frac{dk_2k_2^2}{2\pi^2}$$
$$\times\quad j_{\ell_1}(r_1k_1)j_{\ell_2}(r_2k_2)B_{\ell_1\ell_2L}(k_1,k_2),$$

One more motivation → BAO reconstruction

- Smooth the density field to filter out high k non-linearities.

$$\delta'(\vec{k}) \to e^{-\frac{k^2R^2}{4}}\delta(\vec{k})$$

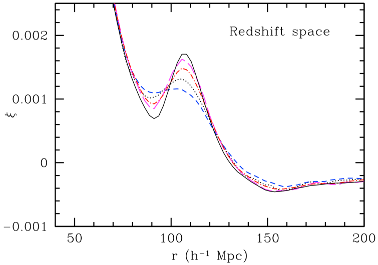
- Solve the Poisson eq. to obtain the gravitational potential

$$\nabla^2\phi = \delta$$

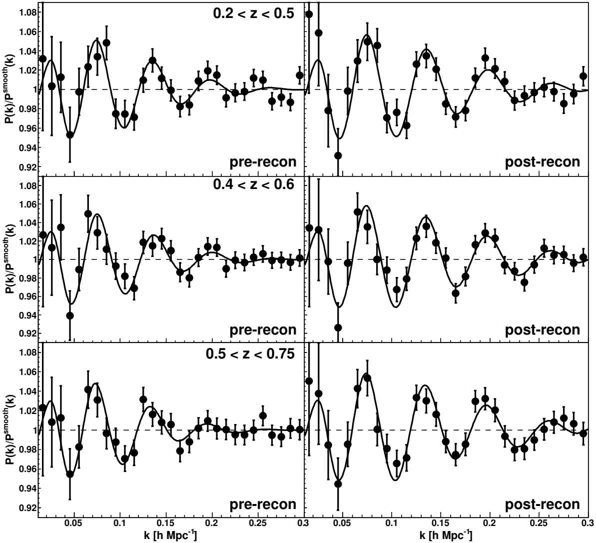
- The displacement (vector) field is given by

$$\Psi = \nabla\phi$$

- Now we calculate the displaced density field by shifting the original particles.

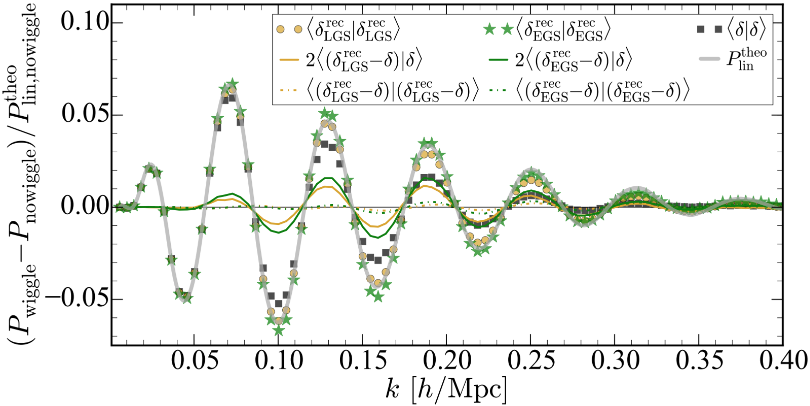


Eisenstein et al. (2007), Padmanabhan et al. (2012)



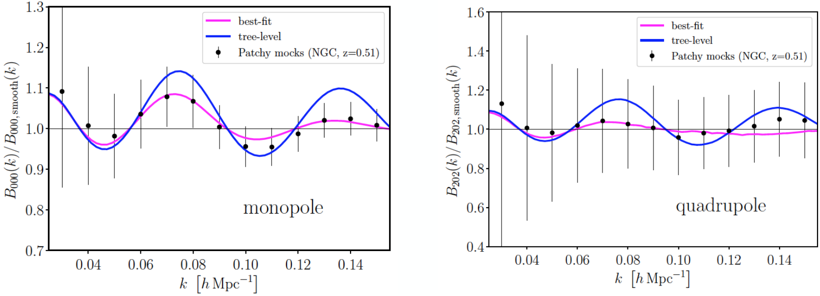
Beutler et al. (2017)

Where does the information come from?



Schmittfull et al. (2015)

BAO in the bispectrum (preliminary)



Sugiyama et al. (in prep.)

Appendix: Accounting for the survey window

We can estimate the survey window very similar to the bispectrum estimator

$$
\begin{aligned}
Q_{\ell_1\ell_2L}(r_1, r_2) = H_{\ell_1\ell_2L} &\sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix} \\
&\times \frac{N_{\ell_1\ell_2L}}{I} \int \frac{d^2\hat{r}_1}{4\pi} y_{\ell_1}^{m_1*}(\hat{r}_1) \int \frac{d^2\hat{r}_2}{4\pi} y_{\ell_2}^{m_2*}(\hat{r}_2) \\
&\times \int d^3x_1 \int d^3x_2 \int d^3x_3 \\
&\times \delta_{\rm D}\left(\vec{r}_1 - \vec{x}_{13}\right) \delta_{\rm D}\left(\vec{r}_2 - \vec{x}_{23}\right) \\
&\times y_L^{M*}(\hat{x}_3)\, \bar{n}(\vec{x}_1)\, \bar{n}(\vec{x}_2)\, \bar{n}(\vec{x}_3).
\end{aligned}
$$

Step 2: Multiply the three-point function with the survey window

$$
\begin{aligned}
&\left\langle \widehat{\zeta}_{\ell_1\ell_2L}(r_1, r_2) \right\rangle \\
&= N_{\ell_1\ell_2L} \sum_{\ell_1'+\ell_2'+L'=\mathrm{even}} \; \sum_{\ell_1''+\ell_2''+L''=\mathrm{even}} \\
&\times \left\{ \begin{matrix} \ell_1'' & \ell_2'' & L'' \\ \ell_1' & \ell_2' & L' \\ \ell_1 & \ell_2 & L \end{matrix} \right\} \left[\frac{H_{\ell_1\ell_2L} H_{\ell_1\ell_1'\ell_1''} H_{\ell_2\ell_2'\ell_2''} H_{LL'L''}}{H_{\ell_1'\ell_2'L'} H_{\ell_1''\ell_2''L''}} \right] \\
&\times Q_{\ell_1''\ell_2''L''}(r_1, r_2)\, \zeta_{\ell_1'\ell_2'L'}(r_1, r_2) \\
&- \quad Q_{\ell_1\ell_2L}(r_1, r_2)\, \bar{\zeta},
\end{aligned}
$$

Appendix: The estimator in detail

The estimator is based on the spherical harmonics expansion proposed in Sugiyama et al. (2017), Hand et al. (2017)

$$\widehat{B}_{\ell_1\ell_2L}(k_1,k_2) = H_{\ell_1\ell_2L} \sum_{m_1 m_2 M} \begin{pmatrix} \ell_1 & \ell_2 & L \\ m_1 & m_2 & M \end{pmatrix}$$
$$\times \frac{N_{\ell_1\ell_2L}}{I} \int \frac{d^2\hat{k}_1}{4\pi} y_{\ell_1}^{m_1*}(\hat{k}_1) \int \frac{d^2\hat{k}_2}{4\pi} y_{\ell_2}^{m_2*}(\hat{k}_2)$$
$$\times \int \frac{d^3k_3}{(2\pi)^3}(2\pi)^3\delta_{\rm D}\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right)$$
$$\times\, \delta n(\vec{k}_1)\,\delta n(\vec{k}_2)\,\delta n_L^M(\vec{k}_3)$$

were y_L^{M*}-weighted density fluctuation

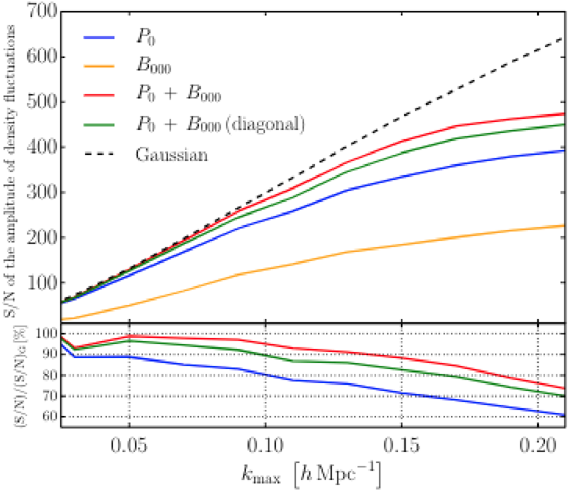
$$\delta n_L^M(\vec{x}) \equiv y_L^{M*}(\hat{x})\,\delta n(\vec{x})$$
$$\delta n_L^M(\vec{k}) = \int d^3x\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\delta n_L^M(\vec{x})$$

and $y_\ell^m = \sqrt{4\pi/(2\ell+1)}\, Y_\ell^m$.

We can apply the same formalism to the three-point function

$$\zeta_{\ell_1\ell_2L}(r_1,r_2) = H_{\ell_1\ell_2L}\sum_{m_1m_2M}\begin{pmatrix}\ell_1 & \ell_2 & L\\ m_1 & m_2 & M\end{pmatrix}\zeta^{m_1m_2M}_{\ell_1\ell_2L}(r_1,r_2).$$

Appendix: Signal to Noise, Recovering the Gaussian information level



Appendix: Relation to other decompositions

Transformation between Scoccimarro (2015) and our decomposition

$$B_{\ell_1\ell_2L}(k_1,k_2) = \frac{N_{\ell_1\ell_2L}H_{\ell_1\ell_2L}}{\sqrt{(4\pi)(2L+1)}}\int\frac{d\cos\theta_{12}}{2}$$
$$\times\left[\sum_M\begin{pmatrix}\ell_1 & \ell_2 & L\\ 0 & -M & M\end{pmatrix}y_{\ell_2}^{-M*}(\cos\theta_{12},\pi/2)\right]\times B_{LM}(k_1,k_2,\theta_{12})$$

Transformation between Slepian & Eisenstein (2017) and our decomposition:

$$B_{\ell_1\ell_2L}(k_1,k_2) = N_{\ell_1\ell_2L}H_{\ell_1\ell_2L}\sum_m(-1)^m\begin{pmatrix}\ell_1 & \ell_2 & L\\ m & -m & 0\end{pmatrix}$$
$$\times\sqrt{\frac{(\ell_1-|m|)!}{(\ell_1+|m|)!}}\sqrt{\frac{(\ell_2-|m|)!}{(\ell_2+|m|)!}}$$
$$\times\int\frac{d\cos\theta_1 d\varphi_{12}}{4\pi}\int\frac{d\cos\theta_2}{2}$$
$$\times\cos(m\varphi_{12})\mathcal{L}_{\ell_1}^{|m|}(\cos\theta_1)\mathcal{L}_{\ell_2}^{|m|}(\cos\theta_2)\times B(k_1,k_2,\theta_1,\theta_2,\varphi_{12})$$